

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 1212

Roll No.

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B. Tech.**(Semester-II) Even Semester Theory Examination, 2012-13****ENGINEERING MATHEMATICS-II**

Time : 3 Hours]

[Total Marks : 100

Note : Attempt questions from each Section as per instructions. The symbols have their usual meaning.

Section-AAttempt *all* parts of this question. Each part carries 2 marks.

2×10=20

1. (a) Verify for exactness the differential equation :

$$y \exp(xy).dx + [x \exp(xy) + 2y] dy = 0.$$

- (b) State the criterion for linearly independent solutions of the homogeneous linear
- n
- th order differential equation.

- (c) Explain ordinary and singular points of a differential equation of the form :

$$a_2(x)y'' + a_1(x)y' + a_0(x)y = 0.$$

- (d) Prove that :

$$J'_0(x) = -J_1(x).$$

- (e) If Laplace transform of
- $f(t)$
- is
- $F(s)$
- , then show that Laplace transform of
- $[e^{at}f(t)]$
- is
- $F(s-a)$
- , where
- a
- is any real number.

- (f) Find :

$$L^{-1}\left[\frac{1}{s^2 - 3s + 3}\right].$$

- (g) Find the partial differential equation which is satisfied by the relation
- $z = c_1xy + c_2$
- , where
- c_1
- and
- c_2
- are constants.

- (h) Give two examples of non-linear partial differential equation of the first order.
 (i) Write telegraph equations.
 (j) Characterize the following partial differential equation into elliptic, parabolic and hyperbolic equations :

$$a \frac{\partial^2 z}{\partial x^2} + 2h \frac{\partial^2 z}{\partial x \partial y} + b \frac{\partial^2 z}{\partial y^2} + 2f \frac{\partial z}{\partial x} + 2g \frac{\partial z}{\partial y} + cz = f(x, y),$$

where a, b, c, h, f, g are constants.

Section-B

Attempt any *three* parts of this question. Each part carries 10 marks. 10×3=30

2. (a) A light elastic string of length l is hung by one end. To the other end are tied successively particles of masses m_1 and m_2 and they produced statical extensions l_1 and l_2 . If T_1 and T_2 are the periods of small oscillations corresponding to the two masses, show that :

$$\frac{T_1^2 - T_2^2}{l_1 - l_2} = \frac{4\pi^2}{g}.$$

- (b) Find the series solution of the differential equation $xy'' + y' + x^2y = 0$.
 (c) Find the solution of the differential equation :

$$\frac{d^2 y}{dt^2} - 2 \frac{dy}{dt} + y = e^t$$

with the initial conditions $y(t) = 0$ and $\frac{dy}{dt} = 1$ when $t = 0$ using Laplace transform.

- (d) Solve the partial differential equation $(p^2 + q^2)y = qz$.
 (e) Solve the one-dimensional heat equation given that $u(0, t) = 0$, $u(l, t) = 0$ and $u(x, 0) = x$, $0 < x < l$, where l is the length of the bar.

Section-C

Attempt any *two* parts from each questions of this Section. Each question carries 10 marks. 5×2×5=50

3. (a) Solve the differential equation :

$$x^2 \frac{d^2 y}{dx^2} + 4x \frac{dy}{dx} + 2y = e^x.$$

- (b) Solve by changing the independent variable :

$$x \frac{d^2 y}{dx^2} + (4x^2 - 1) \frac{dy}{dx} + 4x^3 y = 2x^3.$$

- (c) Solve the system :

$$\dot{x}(t) = y, \dot{y}(t) = -x, \quad x(0) = 0, y(0) = 0.$$

4. (a) Prove that :

$$J_{-n}(x) = (-1)^n J_n(x).$$

- (b) Prove that :

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n.$$

- (c) Prove that :

$$\int_{-1}^1 P_n^2(x) dx = \frac{2}{2n+1}$$

5. (a) Using Laplace transform, evaluate $\int_0^\infty \frac{e^{-t} \sin t}{t} dt$.

- (b) Find the inverse Laplace transform of $\cot^{-1}\left(\frac{s+3}{2}\right)$.

- (c) State convolution theorem and hence evaluate $L^{-1}\left\{\frac{s}{(s^2+1)(s^2+4)}\right\}$.

6. (a) Obtain Fourier series of the function :

$$f(x) = \begin{cases} x, & -\pi < x < 0 \\ -x, & 0 < x < \pi \end{cases}$$

- (b) Find the half range cosine series of the function $f(x) = x \sin x, 0 < x \leq \pi$.

- (c) Solve $(D^2 - 2DD' + D'^2)z = \sin x$.

7. (a) Solve by the method of separation of variables :

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial y} + 2u,$$

$$u(0, y) = 0 \text{ and } \frac{\partial u}{\partial x}(0, y) = 1 + e^{-3y}.$$

- (b) A string is stretched and fastened to two points distance l apart. Find the displacement $y(x, t)$ at any point at a distance x from one end at time t , given that :

$$y(x, 0) = A \sin\left(\frac{2\pi x}{l}\right).$$

- (c) Using the transformations $u = x - ct$ and $v = x + ct$, transform the equation :

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

to the form $\frac{\partial^2 y}{\partial u \partial v} = 0$. Hence show that $y(x, t) = f_1(x - ct) + f_2(x + ct)$.